Particle diffusion in a whistler-like electromagnetic noise

Roch Smets, Gérard Belmont*, Nicolas Aunai*, Laurence Rezeau**

* Laboratoire de Physique des plasmas, UMR 7648, Ecole Polytechnique, route de Saclay, 91128 Palaiseau

Mots Clés : Diffusion, Turbulence - Key Words : Diffusion, Turbulence

Abstract

We study the diffussion process (in position space) of charged particles in an electromagnetic noise. We use self-consistent hybrid electromagnetic code to produce a wide-band noise exhibiting the same properties as the whistler mode. We build the diffusion coefficients, and show that the obtained values are larger than the one predicted by analytical models. The electric fiels (generally not considered in analytical models) is supposed to play in important role by non-linearly coupling with the magnetic field.

Nous étudions la diffusion (dans l'espace des positions) des particules chargées dnas un bruit électromagnétique. Nous utilisons un code hybride auto-cohérent pour produire un bruit large bande ayant des propriétés similaires au mode de sifflement. Nous reconstruisons les coefficients de diffusion et montrons que les valeurs obtenues sont supérieurs à celles prédites par les modèles analytiques. Nous supposons que le champ électrique (générallement non considéré dans ces modèles analytiques) joue un rôle important en se couplant non-linéairement au champ magnétique.

1 - Introduction

The problem of charged particles motion in an irregular electromagnetic field is of importance in various fields of plasma physics : understand the cosmic ray radiations in an irregular interstellar magnetic field (see e.g. [1]), the problem of particle confinement in a tokamak with destroyed magnetic surfaces (see e.g. [2]) or the filling up of the magnetosphere (specifically the Low-Latitude-Boundary-Layer, see e.g. [3]) with particles coming from the solar wind when magnetic reconnection is ineffective. The problem is that simple : how effective is the charged-particle transport depending on the characteristics of the electromagnetic perturbations. It is now understood that electromagnetic noise has consequences for the particle diffusion (in real space) and particle scattering (in velocity space), and that both effects have connections (see e.g. [4]).
In the framework of turbulence, various analytical models have been developed (see e.g. [1], [3], [5], [6]), as well as numerical simulations (see e.g. [4], [5], [6]). Of course, the complexity of the models grows with their accuracy. They generally invoke the particle velocity, the mean magnetic field, and the characteristics of the turbulence like the slope of the spectrum, the

associated energy and the mean free path of the parallel scattering. These models have been conforted with test-particle simulation where a bunch of particles are followed in a prescribed magnetic turbulence to compute a statistical diffusion coefficient.
The nature of the waves is not always considered, and in most of these studies, only the magnetic component is believed to play a role. We thus present original calculations of particle trajectories in a self-consistent electromagnetic hybrid code (see also [7]). We will first introduce the numerical method used, and the way we rebuild the perpendicular diffusion coefficient. Then, we will investigate the nature of the electromagnetic perturbations that develop and diffuse the particles. The last part will focus on the results and comparison with existing models.

2 - Numerical method

We use for this work a self-consistant hybrid code : Protons are computed as macro-particles, electrons are followed as a massless fluid, and electromagnetic fields are calculated under Darwin approximation (neglecting the transverse component of the displacement current see [8] for details on the numerical scheme). The code is 2D in real space ($X-Y$ plan) and 3D in velocity space. We set as initial condition a uniform $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$ magnetic fieldi ($B_0 = 1$), and 2 beams of opposite drift velocity in the X direction to generate the electromagnetic perturbations. As the main magnetic field is in the invariant direction, the particle diffusion is not switched off because of artificial numerical invariant as pointed out by [9].
The perpendicular Fokker-Planck diffusion coefficient $\kappa_\perp$ is defined as

$$\left\langle \frac{\delta y^2}{\delta t} \right\rangle$$

where δy is the drift in Y during δt and the mean value is calculated over the particles. This coefficient quantifies the ability of a particle to separate from the magnetic field line where it was tightened initially. Small values of $\kappa_\perp$ means that particles behave in the magnetic field like beads on a string.
The value of δy is not as straightforward to calculate as in particle-test simulations : because of the self-consistant developing electric field, the field lines are drifting. Because there is no non-ideal effects, the magnetic field is frozen in the plasma. δy is thus the net drift of the particles in the Y direction minus the net drift of the fluid (computed locally for each particles). With this care, there is no trail of the advection in the value of $\kappa_\perp$. FIG. 1 displays $\langle \delta y^2 \rangle$ depending on time. The slope is non-ambiguous, and gives the value of $\kappa_\perp$ depicted by dashed line.
As the plasma is magnetized, the particle gyromotion appears at the gyrofrequency $\omega_C = qB/m$ in FIG. 1. Through time, as the magnetic noise is developing and the gyrophases are mixing, this frequency is smoothed-out. It is not reported here, but the probability distribution functions of both the particle net drift δy and magnetic noise B_z show nice gaussian profiles, excluding any superdiffusion processes as suggested by [3].

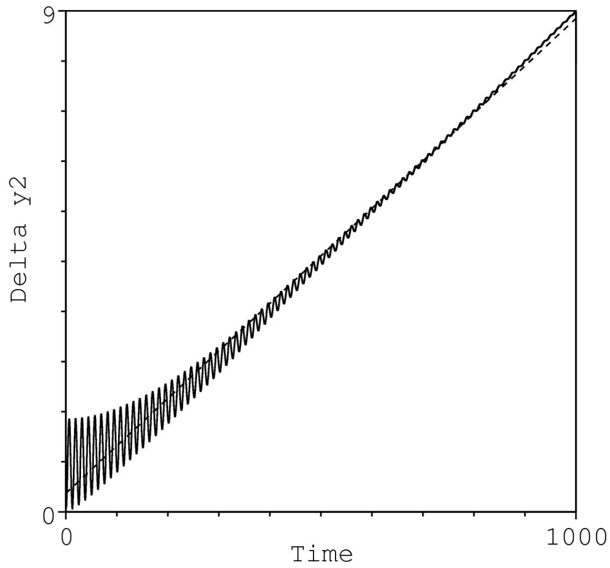

FIG. 1 - $\langle \delta y^2 \rangle$ depending on δt. The slope gives $\kappa_\perp$.

Between different simualtions, we tune the net drift of the beams. As its value increases, the level of both electric and magnetic noise also increases. Furthermore, as this initial configuration is not an equilibrium one, the initial 2 beam distribution functions merge in a resulting Maxwellian distribution with a larger perpendicular temperature $T_\perp$.

3 - Nature of the electromagnetic noise

The time evolution of the plasma parameters (density, velocity and temperatures) and the electric and magnetic perturbation are strongly varying at the begining of the simulations, but get quite stationary values very quickly ($t \sim 20$) compared to the total time ($t = 1000$) of the simulations. In such a stationnary condition, the values of $\langle \delta E^2 \rangle$ and $\langle \delta B^2 \rangle$ is only depending on the value of the initial drift. We had 9 simulations with drift velocity V_d varying between 0 and 4 and a step of 0.5. Figure 2 represent the value of $\langle \delta E^2 \rangle^{1/2}$ depending on $\langle \delta B^2 \rangle^{1/2}$.

The constant slope of $\delta E/\delta B$ is the phase velocity in the case of a monochromatic wave. In the present case, this is the mean value of the phase velocities associated to the different modes, weighted with their amplitude in Fourier space. Parametric studies showed that this slope depends linearly on B_0, n_0^{-1} and q^{-1}. This would suggest that the waves are of whistler type, for which

$$\frac{\omega}{k} = kc^2 \frac{\omega_C}{\omega_P^2} = \frac{k}{\mu_0 q} \frac{B}{n}$$

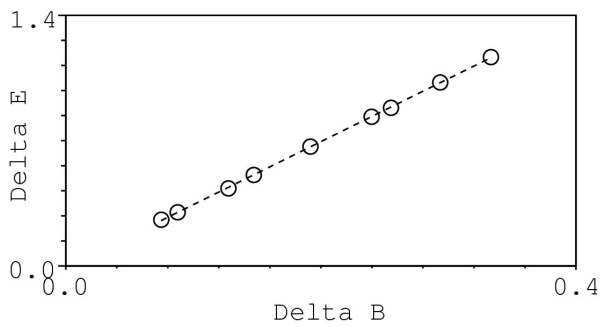

FIG. 2 - $\langle \delta E^2 \rangle^{1/2}$ depending on $\langle \delta B^2 \rangle^{1/2}$.

In the present case, wide band waves can develop. Looking at the spectrum (not reported here), it is wide, noisy, but quite flat. Noting k_l the wave number, $k_l = 2\pi l/L$ for l varying between $-N/2$ and $N/2$ (L is the lenght of the box in the Y direction and N the number of grid points). The phase velocity is proportional to k and the sum over k gives

$$2 \frac{1}{N} \frac{2\pi}{L} \frac{N/2(N/2+1)}{2} = \frac{\pi(N+1)}{2L}$$

In the left-hand side of the above equation, the first $1/N$ is an average (because the spectrum is flat) and factor 2 comes from the symmetric role of positive and negative k. TAB. 1 show the numerical and analytical (with the above expression) values of the slopes for 3 series of runs, varying the ratio N/L. The agreement is very good.

	numerical	analytical
runs (a)	3.74	3.97
runs (b)	2.55	2.64
runs (c)	2.02	1.98

There is just one cloud in this nice picture : the waves in our calculations are propagating in the $X - Y$ plan, perpendicular to the main magnetic field B_0. It can thus not be the parallel whistler mode. But at this stage, it has all the properties of this mode, that is why we call it a whistler-like electromagnetic noise. Furthermore, computing the Z component of $(\mathbf{k} \times \mathbf{E})/\mathbf{B}$ in the Fourier space, one gets the pattern depicted in FIG. 3.

Because of the Maxwell-Faraday equation, this is the dispersion equation $\omega(k)$ of the wave. It exhibit at large k a parabolic shape like the whistler mode.

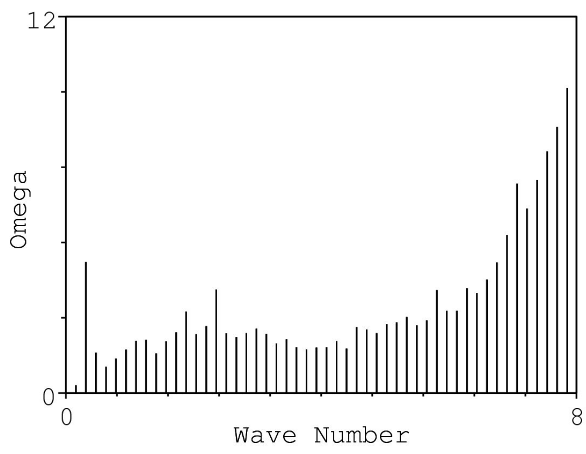

FIG. 3 - $(\mathbf{k} \times \mathbf{E})/\mathbf{B}$ depending on k.

4 - Numerical results

A comparison with different models is not that easy for different reasons : the system is not Hamiltonian (particle energy is varying because of electric work), the geometry is different (invariant in the Z direction), and the population is Maxwellian with a Gaussian distribution of particles velocites. As a base for comparison, we will use the simple analytical model [9], because we can give a simple explanation for it : because of noise in the magnetic field, the Larmor radius is not constant during a cyclotron turn. If a particle drifts of a quantity $\delta\rho_L$ during τ_C, the associated diffusion coefficient is

$$D_\perp = \left\langle \frac{\delta y^2}{\tau_C} \right\rangle$$

The average has to be done on a large number N of cyclotron turns and for all particles. It is thus a temporal and spatial average. If during one cyclotron turn the particles drift from a quantity h, the particle will drift of $\sqrt{N}h$ during N cyclotron turns. The temporal average thus gives the same expression than the spatial one. Assuming half particles are drifting with $\delta y > 0$ and half with $\delta y < 0$, with $\delta\rho_L^2 = \delta x^2 + \delta y^2$ and $\langle \delta x^2 \rangle = \langle \delta y^2 \rangle$, one gets

$$D_\perp = \left\langle \frac{\delta\rho_L^2}{4\tau_C} \right\rangle$$

Using $\dfrac{\delta\rho_L}{\rho_L} = \dfrac{\delta B}{B}$, $\tau_C = \dfrac{m}{eB}$ and $\rho_L = \dfrac{mV_\perp}{eB}$

$$D_\perp = \left\langle \frac{mV_\perp^2}{4eB}\left(\frac{\delta B}{B}\right)^2 \right\rangle$$

With relation $\langle mV_\perp^2 \rangle = k_B T_\perp$, we reach half the value provided by [9] :

$$D_\perp = \frac{k_B T_\perp}{4eB}\left\langle \frac{\delta B^2}{B^2} \right\rangle$$

FIG. 4 displays the numerical values of $\kappa_\perp$ depending on its analytical values given by the above model (open circles). The dotted line indicates the angle bisector of the axes. It is clear that the numerical values obtained are far larger than the analytical predicted ones. But in the present case, one has to consider the self-consistant electric field.

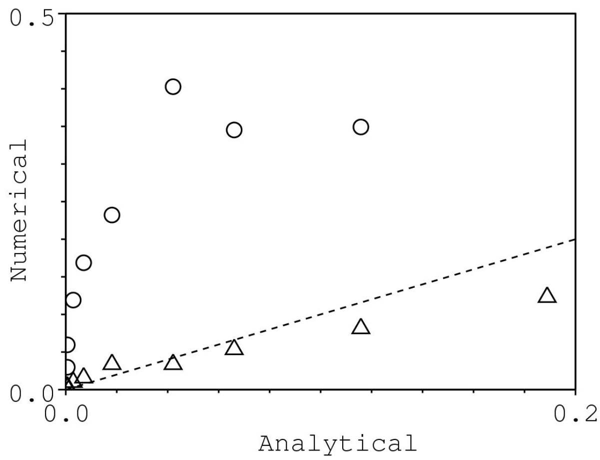

FIG. 4 - Numerical values of $\kappa_\perp$ depending on its analytical values (open circles). The values given by the open triangles are the one obtained with test-particles diffusing in the same electromagnetic noise.

We thus had another set of runs with the same parameters, except that the electric field perturbations were removed (but we considered the same magnetic noise). The obtained results are depicted with open triangles. Our purpose is to emphasize the role of the electric field, generally neglected in test-particle studies. There are two main conclusions : the electric field plays an important role in the diffusion process, and if we only consider the electric perturbation and remove the magnetic perturbation (not depicted here), the associated diffusion coefficient is very small.

5 - Conclusions

We study the particle diffusion in real space because of electromagnetic perturbations in a magnetized plasma. With whistler-like electromagnetic noise and a flat spectrum (wide band noise), we computed the Fokker-Planck diffusion coefficient. We show that analytical model underestimates the diffusion process. We put forward the importance of the electric field, and more specifically, its nonlinear coupling with the role of the magnetic perturbation. At the magnetopause, the Cluster II mission usually observe a value of $\langle \delta B^2/B^2 \rangle \sim 0.15$ (see e.g. [11]). With these values, we reach an analytical value of about 0.4 in the diffusion coefficient. As pointed by [12], the value 0.1 is the threshold to fill-up the Low-Lattitude Boundary Layer. The importance of diffusion process at the magnetopause should then be re-evaluated.

References

[1] Jokipii, J. R., Cosmic-ray propagation. 1. Charged particles in a random magnetic field, Ap. J., 146, 480, 1966

[2] Rechester, A. B. and Rosenbluth, M. N., Electron heat transport in a Tokamak with destroyed magnetic surfaces, Phys. Rev. Lett., 40, 38, 1978

[3] Treumann, R., Theory of super-diffusion for the magnetopause, Geophys. Rev. Lett., 24, 1727, 1997

[4] Minnie, J., W. H. Matthaeus, J. W. Bieber, D. Ruffolo and R. A. Burger, When do particles follow magnetic field lines, J. Geophys. Res., 114, A01102, 2009

[5] Bieber, J. W. and W. H. Matthaeus, Perpendicular diffusion and drift at intermediate cosmic-ray energies, Ap. J., 485, 644, 1997

[6] Matthaeus, W. H., G. Qin, J. W. Bieber and G. P. Zank, Nonlinear collisionless perpendicular diffusion of charged particles, Ap. J., 590, L53, 2003

[7] Winske, D and N. Omidi, Diffusion at the magnetopause : hybrid simulations, J. Geophys. Res., 100, 11,923, 1995

[8] Winske, D. and K. B. Quest, Electromagnetic ion beam instabilities - Comparison of one and two-dimensional simulations, J. Geophys. Res., 91, 8789, 1986

[9] Jokipii, J. R., J. Kóta and J. Giacalone : Perpendicular transport in 1- and 2-dimensional shock simulations, Geophys. Res. Lett., 20, 1759–1761, 1993

[10] Tsurutani, B. T. and R. M. Thorne, Diffusion processes in the magnetopause boundary layer, Geophys. Res. Lett., 9, 1247, 1982

[11] Phan, T. D., C. E. Escoubet, L. Rezeau, R. A. Treumann, A. Vaivads, G. Paschmann, S. A. Fuselier, D. Attie, B. Rogers, B. U. O. Sonnerup, Magnetopause process, Space Sci. Rev., 118, 367, 2005

[12] Sonnerup, B. U. O., Theory of the low-latitude boundary layer, J. Geophys. Res., 85, 2017, 1980