Shell models and the possibility of application to fusion plasmas

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1 Introduction

We present the results of a detailed study of applicability of cascade models, be it in the form of shell models, or in the form of differential approximation models, to fusion plasmas. There exists various difficulties associated with the plasma medium. Its intrinsic excitability, the role of meso-scales and its natural anisotropy can be mentioned. In this work, we derive a set of shell models, that are tailored for applicability to fusion plasmas. We suggest this as a useful method for understanding the nonlinear tendencies instead of direct prediction.

2 Shell model for drift waves

Here we review the basic idea behind shell models and give the simplest possible example in the form of a shell model of the Hasegawa-Mima system [3]. This allows a demonstration of the basic idea as well as the derivation of a differential approximation model.

3 Potential Vorticity Conservation

Here we will discuss the role of potential vorticity in fusion plasmas [4, 1]. Using a simple reduced system, we will show that the concept of potential vorticity, used prominently in geophysical fluid dynamics is a useful concept also for fusion plasmas.

3.1 Generalized shell model in the form of Potential Vorticity evolution

A simple shell model can be formulated in the form of potential vorticity evolution, using the basic idea that the potential vorticity is approximately conserved in fusion plasmas. This allows the reduction of very complex nonlinear system to a set of, relatively simple, coupled ordinary differential equations. In the limit of local interactions, the system leads to solutions akin to 2D Kraichnan-Kolmogorov spectra. In contrast, when the non-local interactions with a single mode dominates (assuming also that the electron response is adiabatic), a solution in the form of $|\widetilde{n}_k|^2 \sim \left|\widetilde{\Phi}_k\right|^2 \propto k^{-3}/\left(1+k^2\right)^2$ is found [2].

3.2 Predator Prey oscillations

When the non-local interactions with a single large scale mode (zonal flows, Geodesic Acoustic Modes etc.) dominate, the above model leads to an interesting dynamical evolution. Since the large scale mode is not driven, it is driven purely by the interactions among fluctuations. However, as it grows, it refracts the the fluctuations to higher wave-numbers, where the dissipation is higher.

Therefore the large scale mode ends up depleting its own source for drive. When this happens, it is also damped (physically by a large scale drag process, modelled as a drag term in the equation for the evolution of the large scale). But since there is constant energy injection, when the large scale mode damps, the fluctuations can grow again. This leads to a phenomenon akin to predator-prey oscillations. At the extreme, these oscillations can be observed using a minimally reduced shell model of two shells and the large scale component.

4 Conclusion

We have studied the feasibility of the application of shell models to fusion plasmas. While their predictive power is limited as a result of lack of direct connection to physics parameters, their qualitative utility is undeniable. We find that due their simplicity they can be implemented rather easily, and can tackle a complex nonlinear problem with some (sometimes strong) assumptions. They can also be used in conjunction with proper linear physics (for instance a linear gyro-kinetic code, which can give the exact form of the energy injection and dissipation) for increased accuracy.

References

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