Goniopolarimetry with Coupled Electric and Magnetic Measurements

Mesures goniopolarimétriques couplées électriques et magnétiques

B. Cecconi [*1], A.-L. Gautier [*2], J.E.S. Bergman [†3], T. Chust [‡4], A. Marchaudon [#5], C. Cavoit [§6], O. Santolìk [%7]

[*] *LESIA, CNRS – Observatoire de Paris – UPMC – Univ. Denis Diderot, 5 Place Jules Janssen, 92190 Meudon, France*
[1] `baptiste.cecconi@obspm.fr`, [2] `anne-lise.gautier@obspm.fr`

[†] *IRF-Uppsala Ångström Laboratory, Lägerhyddsv. 1, Uppsala, Sweden*
[3] `jb@irfu.se`

[‡] *LPP, CNRS – Ecole Polytechnique – UPMC – Univ. Paris Sud, Route de Saclay, 91128 Palaiseau Cedex, France*
[4] `thomas.chust@lpp.polytechnique.fr`

[†] *IRAP, CNRS – Univ. Paul Sabatier, 9 Avenue du Colonel Roche, BP 44346, 31028 Toulouse Cedex 4, France*
[5] `aurelie.marchaudon@irap.omp.eu`

[§] *LPC2E, CNRS – Univ. d'Orléans, 3A, Avenue de la Recherche Scientifique, 45071 Orlans cedex 2, France*
[6] `cavoit@cnrs-orleans.fr`

[%] *Department of Space Physics, Institute of Atmospheric Physics AS CR, Bocni II/1401, 14131 Praha 4, Czech Republic*
[7] `os@ufa.cas.cz`

***Abstract*—Goniopolarimetric techniques (also known as "Direction-Finding") are making use of either electric or magnetic component measurements to deduce the observed electromagnetic wave parameters: its flux density, polarization state and direction of k-vector. Radio receivers such as Cassini/RPWS/HFR or STEREO/Waves provide goniopolarimetric measurements and thus allow the scientists to obtain rich data products that lead to high level science results. However, in some cases, these measurements cannot resolve specific ambiguities: (a) when close to the source it is not possible to derive the sense of propagation; (b) when close to the wave mode cutoff, the transverse propagation assumption (necessary with electric measurements) is not valid any more. Future radio receivers will include simultaneous electric and magnetic component measurements. We explore here how this instrumental setup provides enough additional information to get correct and accurate results in the two problematic situations presented above.**

Index Terms*—Radioastronomy, Goniopolarimetry / *Radioastronomie, Goniopolarimétrie

I. INTRODUCTION

Space based radio observatories are necessary to observe radio waves with frequencies below $10\,MHz$ because of the Earth ionospheric cutoff, but the constraints on space-borne instrumentation are drastically limiting the capabilities of these observatories (especially in terms of angular resolution) compared to their ground-based brothers. As discussed in [**?**], "Goniopolarimetry" (also referred to as "Direction Finding" in the literature) is a using spectral power measurements to retrieve the waves parameters. These techniques use electric (resp. magnetic) sensors that are short dipoles (resp. search coils) with small dimensions (usually a few m for electrical antennae and a few 10's of cm for search coils) compared to the observed wavelengths.

The two main factors used to select which component (electric or magnetic) of the wave is measured, are the following: (i) sensitive electric sensors in the radio range are easier to build than magnetic ones; (ii) the plasma quasi-thermal noise and the impact noise dominate the electric components of local waves in the low frequency range. Hence, magnetic search coils have been developed to observe plasma waves, whereas electric antennae are used in the radio range. There have been electric and magnetic observations in the early exploration of the terrestrial Auroral Kilometric Radiation (AKR) with the IMP-6 spacecraft (see Fig. 2 of [1]). These observations at 178 kHz showed the electromagnetic nature of AKR. Thanks to recent technology developments [2], low noise sensitive preamplifiers for magnetic sensors are now available, which allows their use in the radio range up to a few 10's of MHz.

II. GONIOPOLARIMETRY

The angular resolution of space based radio sensors is intrinsically very poor. However, the wave parameters (direction of wave vector, polarization degree and flux) can be recovered using goniopolarimetry (GP) [**?**]. GP has been successfully used on Cassini/RPWS [3] data observing SKR (Saturn Kilometric radiation), see e.g., [4], [5], [6], [7].

As shown on Fig. 2 of [8], the receiving pattern of an antenna (i.e., the gain of the antenna, depending on the direction of arrival of the observed wave) changes with the sensor length to wave length ratio. For wavelengths much larger than the antenna length, the receiving pattern of an electric (or

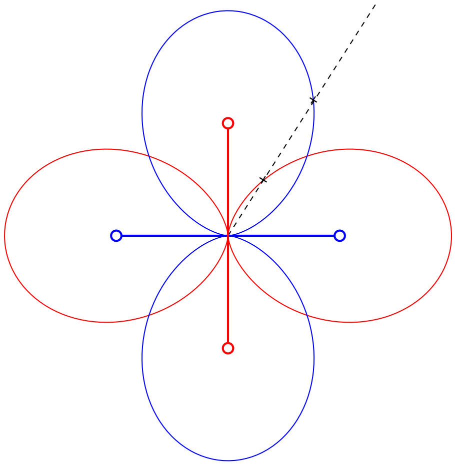

Fig. 1. Simplified Goniopolarimetric measurement. Two crossed electric dipoles (blue and red) with their associated antenna pattern. Comparing the measured power induced by a wave coming in the direction of the dashed line will provide wave direction of arrival. Real GP instrumentation includes cross-correlation of signals measured on each antenna, and thus provides phasing measurements that permit the reconstruction of the polarization state of the wave.

magnetic) sensor is that of a perfect electric dipole (or loop). This wavelength range is called the quasi-static range (or short antenna range) and is defined as: $\lambda \gg L$ where L is the size (length or diameter) of the sensor. GP techniques have been developed for measurements in the quasi-static range. The instantaneous voltage induced by an electromagnetic wave passing around an electric dipole (in its quasi-static range) is:

$$V(t) = \mathbf{A}.\mathbf{E}(t) \tag{1}$$

where $\mathbf{A}$ is the effective antenna vector, and $\mathbf{E}$ is the electric field of the wave. Because of telemetry constraints, the instantaneous waveform of the sensed voltages are not recorded at radio frequencies. Conversely, we store auto- and cross-correlations of signals measured on several sensors with different orientations. Using the fact that the Fourier transform of a correlation is a spectral power density (Wiener-Khinchin theorem), we obtain the spectral power P_{ij} as follows:

$$P_{ij}(f) = \left\langle V_i(f) V_j^*(f) \right\rangle = \langle (\mathbf{A}_i.\mathbf{E})(\mathbf{A}_j.\mathbf{E})^* \rangle \tag{2}$$

where $\langle \cdots \rangle$ means averaging on a time larger than the wave period, and the i and j subscripts refer to two antennae. In case $i = j$, the measurement is an auto-correlation (i.e., the power) of the signal sensed on the antenna, whereas when $i \neq j$, it is a cross-correlation of the signals on two antennae. This cross-correlation contains phase information that is used to retrieve the wave parameters.

Figure 1 shows a simplified goniopolarimetric set-up with two orthogonal electric dipoles. From this figure, we can see that the ratio of the power measured on each antenna can lead to the determination of the wave direction of arrival. This simplified figure realistically illustrates the case of an unpolarized wave, propagating in the plane formed by the antennae and in a non-magnetized vacuum. In a general way, GP inversions use auto- and cross-correlations to retrieve the polarization degree, the polarization plane and the intensity of the electric field component $\mathbf{E}$. When assuming transverse propagation ($\mathbf{k}.\mathbf{E} = 0$), we can easily obtain the direction of the wave vector $\mathbf{k}$.

A. Spectral matrix

Several GP inversions have been developed, depending on the type of measurement, and on the number of simultaneously measured components. When all components are available, or in a theoretical case, the spectral matrix formalism (see [9] and references therein) is well adapted.

In the three dimensional space, the spectral matrix (or coherence matrix) of the electric component of a radio wave is defined as follows:

$$\mathcal{S}_{EE} = \frac{1}{2Z_0} \left\langle \mathbf{E}(t).\mathbf{E}(t)^H \right\rangle \tag{3}$$

with Z_0 the impedance of free space, and ^{H} indicating the hermitian transform (conjugate transposed). With this formalism, the measurements matrix $\mathcal{P}_{EE}$ writes $\mathcal{P}_{EE} = \mathcal{A}.\mathcal{S}_{EE}.\mathcal{A}^H$ where the matrix $\mathcal{A}$ is composed of the three effective antenna vectors. The GP analysis consists here in inverting $\mathcal{P}_{EE}$, knowing $\mathcal{A}$. When the measurements are made on three antennae (with effective directions not restricted in a plane, and at best mutually orthogonal), it is possible to simply invert the system. The measurement matrix $\mathcal{P}_{EE}$ is a 3×3 matrix containing the nine P_{ij} measurements on the three electric antennae. If the antennae directions are not all in the same plane, the matrix $\mathcal{A}$ is invertible, and we obtain:

$$\mathcal{S}_{EE} = (\mathcal{A}^H)^{-1}.\mathcal{P}_{EE}.(\mathcal{A})^{-1} \tag{4}$$

There are only 6 independent measurements among the nine values of matrix $\mathcal{P}$: three $\mathcal{P}_{ii}$ autocorrelations, and three $\mathcal{P}_{ij}$ cross-correlations. This is simply deduced from the fact that $\mathcal{P}_{ij}^* = \mathcal{P}_{ji}$. Hence, an ideal three channel GP radio receiver records 6 measurements instantaneously. These 6 measurements are in reality stored as 9 real values because the cross-correlations are complex numbers, and are each represented by two real numbers.

B. Goniopolarimetry Limitations

The three main limitations of GP measurements and inversions: (i) assuming transverse propagation is needed to infer the wave vector direction with solely electric measurements; (ii) when measuring only the electric (or magnetic) field components, we get the direction of the wave vector, but not its sense; (iii) the Poynting vector provides a direction of propagation of energy (i.e., the ray path), it can not be directly inferred from GP measurements, and can be different of the direction of the wave vector. Another limitation of GP (not studied here) is the point source (or plane waves) assumption.

III. PROPAGATION MODES

In the frame of the magneto-ionic theory (see, e.g., [10], [11]), free-space electromagnetic waves propagates in a cold and magnetized plasma on two characteristic modes: the L-O and R-X modes, which stand respectively for left-handed ordinary and right-handed extraordinary modes. There are two other trapped modes: the Whistler-mode and the Z-mode, which correspond respectively to L-O and R-X modes. In this study, we only study free space propagating modes, but the same study can be undertaken for the trapped modes.

The propagation modes are the solutions of the dispersion equation that relates the wave pulsation ω to its wave vector k. This relationship is in general given in the form of the refractive index $n(\omega)$, which is equal to ck/ω by definition, where k is a function of ω. However, the determination of the wave vector direction is only part of the analysis. The direction of propagation of the energy (i.e., the direction of arrival of the ray) is given by the mean Poynting vector $\langle \mathbf{\Pi} \rangle$. Determining the direction of $\langle \mathbf{\Pi} \rangle$ is thus essential for goniopolarimetric analyses in regions where $n \neq 1$. When $n = 1$, $\langle \mathbf{\Pi} \rangle$ is aligned with $\mathbf{k}$.

In a general case, the electromagnetic waves are propagating obliquely with the ambient magnetic field. The dispersion equation is then given by the Appleton-Hartree equation [12].

IV. COUPLED ELECTRIC AND MAGNETIC MEASUREMENTS

A. *Electromagnetic Spectral Matrix*

The spectral matrix $\mathcal{M}(\mathbf{E}, c\mathbf{B})$ is defined as follows:

$$\mathcal{M}_{EB} = \begin{bmatrix} \mathcal{S}_{EE} & \mathcal{S}_{EB} \\ \mathcal{S}_{BE} & \mathcal{S}_{BB} \end{bmatrix} \tag{5}$$

with $\mathcal{S}_{EE}$ defined as in Eq. (3), and $\mathcal{S}_{EB} = c/2Z_0 \left\langle \mathbf{E}.\mathbf{B}^H \right\rangle$, $\mathcal{S}_{BE} = c/2Z_0 \left\langle \mathbf{B}.\mathbf{E}^H \right\rangle$, $\mathcal{S}_{BB} = c^2/2Z_0 \left\langle \mathbf{B}.\mathbf{B}^H \right\rangle$. We see that $\mathcal{S}_{BE} = \mathcal{S}_{EB}^H$. Hence, only one of these two matrices can be retained. We choose to keep $\mathcal{S}_{EB}$, for the following developments. The components of the Poynting vector are given by the off-diagonal elements of $\mathcal{S}_{EB}$.

B. *Sense of propagation*

It is possible to obtain the upper-left 3×3 matrix ($\mathcal{S}_{EE}$) with classical GP techniques, by inverting the corresponding $\mathcal{P}_{EE}$ measurement matrix. In case we have one supplementary axis (one magnetic in case of three electric, or one electric in case of three magnetic), and measure the cross-correlation between one electric and one magnetic sensor, we get only one column of $\mathcal{S}_{EB}$. A simple analysis shows that comparing the sign of its values to that of $\mathcal{S}_{EE}$ provides the sense of propagation of the wave. This solution requires only four sensors coupled with a four-channel radio receiver computing the full set of auto- and cross-correlations (i.e., 4 autocorrelations and 6 cross-correlations, stored into 16 real values).

C. *Near cut-off propagation*

When close to the wave's propagation cut-off surface, the refractive index goes down to zero. The radio wave's propagation is thus strongly influenced by the local plasma conditions. We assume that the wave is propagating in a cold magnetized plasma, which is homogeneous at the wavelength scale. In the general case, if the local magnetic field direction and strength, as well as the local plasma density are known, it is possible to compute the angle θ between the wave vector and the local magnetic field direction for electric measurements (even in case of a non-tranverse propagation). The GP inversion provides the orientation of the electric field polarization plane. The magnetic component is in a plane perpendicular to the wave vector, and the single magnetic component measurement allows to constrain the magnetic field polarization ellipse. Then the Poynting vector direction determination is straightforward. In the case of an unknown or poorly determined plasma density, it is still unclear whether it is possible to invert the system and retrieve the wave vector sense and direction, as well as the Poynting vector direction.

V. CONCLUSION

This paper presents the gain of an additional magnetic component to a classical three axis electric radio receiver system. The instrumental setup is planned for the future JUICE/RPWI (Jupiter Icy Moon Explorer/Radio and Plasma Wave Instrument). During the last phases of that mission, the spacecraft will fly very close to Ganymede, in its ionosphere, where the radio waves emitted by Jupiter will be strongly refracted. This requires the correct determination of the wave and Poynting vectors in order to either either local plasma properties or the wave properties outside Ganymede's ionosphere.

ACKNOWLEDGMENT

The french author acknowledge CNES (Centre National d'Etudes Spatiales) for supporting JUICE/RPWI studies.

REFERENCES

[1] D. A. Gurnett, "The Earth as a Radio Source: Terrestrial Kilometric Radiation," *J. Geophys. Res.*, vol. 79, no. 28, pp. 4227–4238, 1974.

[2] C. Cavoit, "Closed loop applied to magnetic measurements in the range of 0.1–50 MHz," *Rev. Sci. Instr.*, vol. 77, no. 064703, 2006.

[3] D. A. Gurnett, W. S. Kurth, D. L. Kirchner, G. Hospodarsky, T. Averkamp, P. Zarka, A. Lecacheux, R. Manning, A. Roux, P. Canu, N. Cornilleau-Werhlin, P. Galopeau, A. Meyer, R. Bostrom, G. Gustafsson, J.-E. Wahlund, L. Åhlen, H. Rucker, H.-P. Ladreiter, W. Macher, L. Woolliscroft, H. Alleyne, M. L. Kaiser, M. Desch, W. M. Farrell, C. Harvey, P. Louarn, P. J. Kellogg, K. Goeth, and A. Pedersen, "The Cassini radio and Plasma wave science investigation," *Space Sci. Rev.*, vol. 114, no. 1–4, pp. 395–463, 2004.

[4] B. Cecconi, L. Lamy, P. Zarka, R. Prangé, W. S. Kurth, and P. Louarn, "Goniopolarimetric study of the revolution 29 perikrone using the Cassini Radio and Plasma Wave Science instrument high-frequency radio receiver," *J. Geophys. Res.*, vol. 114, no. A03215, 2009.

[5] G. Fischer, B. Cecconi, S.-Y. Ye, P. Zarka, A. Lecacheux, L. Lamy, W. S. Kurth, and D. Gurnett, "Elliptical polarization of Saturn Kilometric Radiation observed from high latitudes," *J. Geophys. Res.*, vol. 114, p. A08216, 2009.

[6] L. Lamy, P. Schippers, P. Zarka, B. Cecconi, C. Arridge, M. Dougherty, P. Louarn, N. André, W. S. Kurth, R. Mutel, D. Gurnett, and A. Coates, "Properties of Saturn kilometric radiation measured within its source region," *Geophys. Res. Lett.*, vol. 37, no. L12104, 2010.

[7] L. Lamy, B. Cecconi, P. Zarka, P. Louarn, W. S. Kurth, J. D. Menietti, and R. L. Mutel, "SKR properties from measurements within its emission region: polarization and modes of emission," *J. Geophys. Res.*, vol. in press, 2011.

[8] B. Cecconi, "Goniopolarimetric techniques for low-frequency radio astronomy in space," in *Observing Photons in Space*, M. C. E. Huber, A. Pauluhn, J. L. Culhane, J. G. Timothy, K. Wilhelm, and A. Zehnder, Eds. ISSI/ESA, Bern, Switzerland, 2010, pp. 263–277.

[9] A. Lecacheux, "Direction Finding of a Radiosource of Unknown Polarization with Short Electric Antennas on a Spacecraft," *Astron. Astrophys.*, vol. 70, pp. 701–706, 1978.

[10] K. Budden, *The propagation of Radio Waves: The Theory of Radio Waves of Low Power in the Ionosphere and Magnetosphere.* New York, USA: Cambridge University Press, 1985.

[11] D. Gurnett and A. Bhattacharjee, *Introduction to Plasma Physics*. Cambridge, UK: Cambridge University Press, 2005.

[12] E. Appleton, "Wireless studies in the ionosphere," *J. Inst. Electr. Eng.*, vol. 71, pp. 642–650, 1932.