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Determination of constitutive parameters for chiral homogeneous metamaterials from transmission and reflection coefficients

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Abstract
In the proposed paper is shown a novel approach of determination of constitutive parameters for bi-isotropic chiral metamaterials. The equivalent model of medium is described with permittivity, permeability and chirality which are needed to be retrieved from the transmission and reflection coefficients of the chiral slab. The electromagnetic coupling in bi-isotropic media leads to handedness and two left and right handed propagation constants. As well this property leads to cross term of transmission coefficient and allows to retrieve constitutive parameters.

Introduction
Metamaterial chiral medium is the bi-isotropic medium which does not give conformance with its mirror and rotation. In special case of bi-isotropic medium when only handedness degree exists the medium is called Pasteur medium and this one is in the interest of presented paper. The handedness describers electromagnetic coupling, cross-coupling between electric and magnetic fields which leads to creation of left and right handed waves propagating inside the medium. Once, a slab of chiral medium is excited with linear polarized plane wave inside the medium are created two waves with different phase velocities, which are respectively left and right handed waves. In the backside of the slab the two waves are combined and create one linear polarized plane wave but rotated in respect to the incident wave [1, 2]. The bi-isotropic media in special condition can provide the negative refraction [1]. However, this property is not possible to find in the natural chiral materials, due to fact of low degree of chirality [3-5] but it can be obtained with MMs like cubic chiral SRRs [4] or lattice of cross rotated multilayer structure [5].

The reciprocal MMs chiral medium can be expressed with equivalent model of medium described by constitutive relation. The equivalent parameters of such bi-isotropic medium (permittivity, permeability and chirality) can be found through measurements of its finite structure. However, the conventional approach [6] of retrieving of constitutive parameters form transmission and reflection coefficients is not valid for such medium, additional unknown of chirality. In the assumption of excitation of chiral medium with circular wave which is as well eigen wave of the medium the method [6] can be applied in the determination of constitutive parameters [2]. Other way, which can be extended beyond circular waves, is excitation with linear polarized waves and due to the handedness of the medium in the measurement can be found a cross polarization transmission term, expressing cross term between two polarization of plane waves TE and TM [7]. The additional measurement coefficient allows to solve the equations system to determine left and right propagation constants and wave impedance of the medium. From the infinite medium is known relation of constitutive parameters and the propagation constants and wave impedance what leads to retrieving approach for bi-isotropic chiral medium.

1. Propagation in infinite chiral medium
In the characterization of medium it is needed to know the relation between propagation constants and related wave impedance with constitutive parameters. For the infinite medium without any source inside it is assumed that characteristic waves are propagating along of one principal axis. The Maxwell's equation and constitutive relation of Pasteur medium [1] can be written in meaner to reduce of the fields components [8] or by derivation of characteristic equation. The obtained propagation constants are different for the right and left handed ($k_{\parallel}$ and k_{Γ}), respectively related with the fields of TM and TE polarizations [1,2,7]. In convection isotropic media those two propagation constants are the same, where in Pasteur media two orthogonal waves are distinguished. With known propagation constants the characteristic waves are described for each and the wave admittance of the medium can be found [1, 7]. From the propagation constant it can be observed that for chiral term bigger then square root of equivalent permittivity and permeability leads to negative refraction.

2. Determination of constitutive parameters form chiral slab

The known relations from infinite structure provides determination of constitutive parameters for given propagation constants and wave admittance, like in [2,7]. However, infinite medium is not practical in measurement setup, thus it is more common to use finite one (slab of chiral medium), similar like in isotropic media [6]. The determination of propagation constants and wave admittance requires to set a particular boundary condition between chiral medium and background medium. The characteristic waves related with propagation constants are expressed by mixture of TE and TM fields components. In the consideration of boundary conditions for semi-infinite medium it is easy to find that due to chirality the fields components are also mixed inside the medium. For more clear notation of the fields mixture it is better to write in matrix notation of boundary conditions as $[\underline{E}_i^{A}] = [\underline{Y}_{tot}]\ [\underline{E}_e]$, where matrix for incident fields and fields inside the medium contains forward and backward TE/TM components [7]. The matrix $[\underline{Y}_{tot}]$ contains the admittance information normalized to the background medium A (assumed as a free space). With an inversion of the one side boundary conditions and application of propagator $[\underline{P}]$ connecting two interfaces of slab, leads to definition of fields relation of MMs slab $[\underline{E}_i^{A}] = [\underline{Y}_{tot}]\ [\underline{P}]\ [\underline{Y}_{tot}]^{-1} = [\underline{E}_i^{B}]$. Form this relation the transmission and reflection matrix are determined and their inversion in term of propagation constants and wave admittance gives:

$$Y = \pm\sqrt{\frac{R_\parallel^2 - 2R_\parallel - T_\parallel^2 - T_{\parallel\Gamma}^2 + 1}{R_\parallel^2 + 2R_\parallel - T_\parallel^2 - T_{\parallel\Gamma}^2 + 1}} \qquad (1)$$

$$e^{jk_\Gamma d} = \frac{T_\parallel Y^2 - T_\parallel \pm \sqrt{\left(T_\parallel^2 - R_\parallel^2 - 2R_\parallel - 1\right)\left(Y^4 + 1\right) + 2\left(R_\parallel^2 - T_\parallel^2 + 1\right)Y^2}}{Y^2 + R_\parallel Y^2 - 2R_\parallel Y - 1} \qquad (2)$$

$$e^{jk_\parallel d} = e^{-jk_\Gamma d}\,\frac{Y^2 + R_\parallel Y^2 + 2R_\parallel Y + R_\parallel - 1}{Y^2 + R_\parallel Y^2 - 2R_\parallel Y + R_\parallel - 1} \qquad (3)$$

, where $T_{\parallel\perp}$ is cross polarized transmission coefficient, $T_\parallel$ and $R_\parallel$ are transmission and reflection for one type of polarization (TE). In the determination are need to distinguished two propagation constants which are related each other ($k_\parallel$ and $k_\perp$ correspond to wave propagation polarized as TE and TM). The correct signs ± in equations (1-2) are determined with causality and passivity, laws which have to be satisfied for any homogeneous medium [9]. With known propagation constants and admittance, the constitutive parameters can be found directly from transmission and reflection coefficients of any Pasteur type media.

3. Conclusion

In presented paper is shown retrieval method of constitutive parameters through propagation constants and wave impedance obtained from transmission and reflection coefficients of homogeneous chiral slab. The property of chiral media, rotation of polarization of travelling wave, leads to additional measurement of cross transmission term. With correct determination of boundary condition for the chiral slab the inversion method of transmission and reflection in term of propagation constants and wave impedance allows to determinate equivalent permittivity permeability and chirality of the medium under investigation.

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